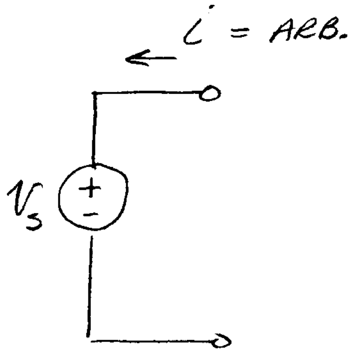
CH. 2: FUNDAMENTALS OF ACTIVE CIRCUIT THEORY

A) NULLATORS AND NORATORS

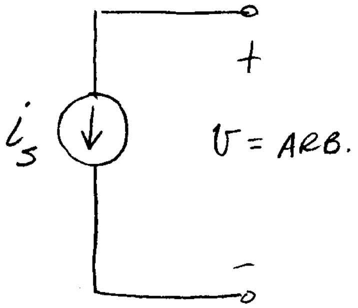
1) INDEPENDENT VOLTAGE SOURCE

- FOR A VOLTAGE SOURCE, THE VOLTAGE IS SPECIFIED AND THE CURRENT IS NOT SPECIFIED (ARBITRARY). THE VALUE OF THE CURRENT IS DETERMINED BY THE CIRCUIT V_S IS CONNECTED TO.



2) INDEPENDENT CURRENT SOURCE

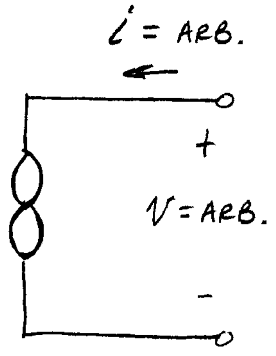
- FOR A CURRENT SOURCE, THE CURRENT IS SPECIFIED AND THE VOLTAGE IS NOT SPECIFIED (ARBITRARY). THE VALUE OF THE VOLTAGE IS DETERMINED BY THE CIRCUIT i_S IS CONNECTED TO.



- IT WOULD SEEM THAT THERE SHOULD EXIST AN ELEMENT WITH AN ARBITRARY CURRENT AND AN ARBITRARY VOLTAGE.

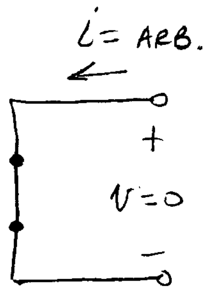
3) NORATOR

- A NORATOR IS A CIRCUIT ELEMENT WHERE NEITHER THE VOLTAGE NOR THE CURRENT IS SPECIFIED FOR THE GENERATOR.



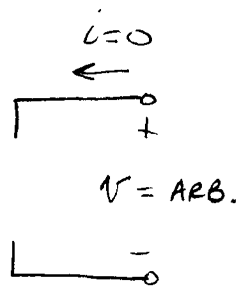
4) SHORT CIRCUIT

- A SHORT CIRCUIT HAS A FIXED VOLTAGE OF ZERO AND AN ARBITRARY CURRENT.



5) OPEN CIRCUIT

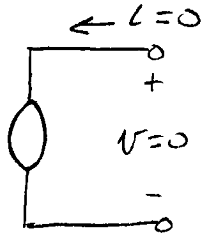
- AN OPEN CIRCUIT HAS A FIXED CURRENT OF ZERO AND AN ARBITRARY VOLTAGE.



- IT WOULD SEEM THAT THERE SHOULD EXIST ANOTHER CIRCUIT ELEMENT IN WHICH BOTH THE VOLTAGE AND CURRENT ARE BOTH ZERO.

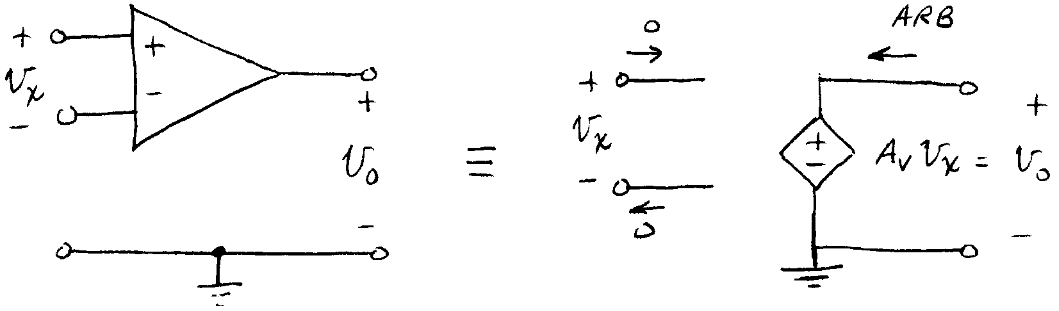
6) NULLATOR

- A NULLATOR IS A CIRCUIT ELEMENT WHERE VOLTAGE AND CURRENT ARE NULLED FOR THE GENERATOR.



7) IDEAL OP-AMP

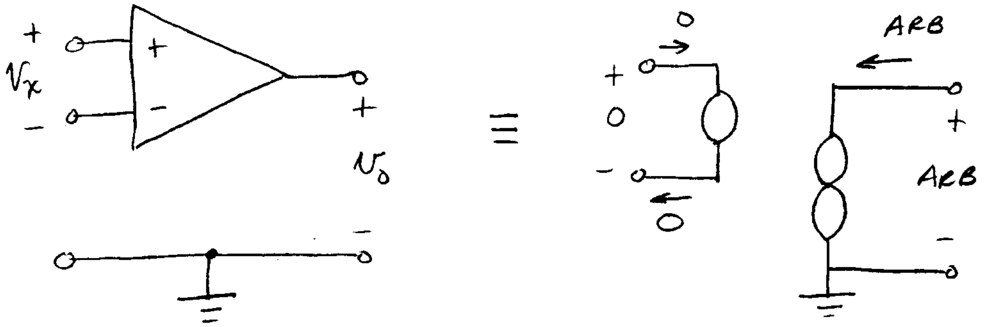
– AN IDEAL OP-AMP IS A VOLTAGE-CONTROLLED VOLTAGE SOURCE WITH INFINITE GAIN.



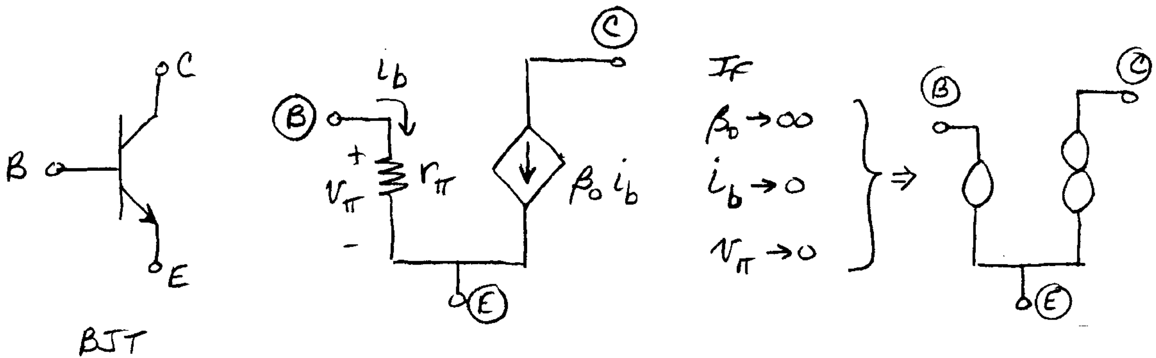
IF v_o IS FINITE (STABLE) THEN

$$A_v v_x = v_o \Rightarrow v_x = \left.\frac{v_o}{A_v}\right|_{A_v \to \infty} \to 0$$

AND $v_o = A_v v_x = (\infty)(0) = ARB.$



8) IDEAL BIPOLAR JUNCTION TRANSISTOR (BJT)



IF
$\beta_o \to \infty$
$i_b \to 0$
$v_\pi \to 0$
$\Rightarrow$

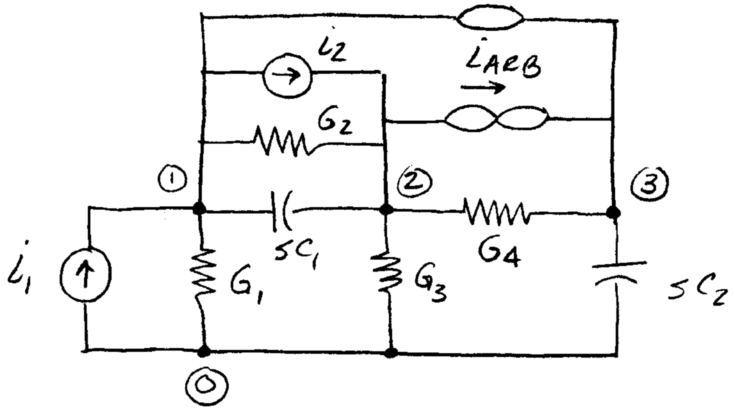
BJT

AC MODEL
(LOW FREQUENCY)

CALLED A
NULLOR

B) NULLATOR - NORATOR NODAL ANALYSIS

- SUPPOSE WE ADD AN IDEAL BJT TO THE CIRCUIT IN CH 1, p1



- DOING THE INSPECTION ALGORITHM

$$\begin{array}{c}\text{①}\\ \text{②}\\ \text{③}\end{array}\begin{bmatrix} i_1 - i_2 \\ i_2 - i_{ARB} \\ i_{ARB} \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \text{①} & \text{②} & \text{③} \end{array} \\ \begin{bmatrix} (G_1+G_2+sC_1) & -(G_2+sC_1) & 0 \\ -(G_2+sC_1) & (G_2+G_3+G_4+sC_1) & -G_4 \\ 0 & -G_4 & (G_4+sC_2) \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

WHERE WE HAVE ACCOUNTED FOR THE EXTRA CURRENT OF THE NORATOR. ALSO NOTE THAT WE HAVE ADDED ANOTHER UNKNOWN, i_{ARB}, TO OUR SET OF UNKNOWNS V_1, V_2, V_3.

- WHAT EFFECT DOES THE NULLATOR HAVE? THE CURRENT IS ZERO SO NO EFFECT TO THE LEFT SIDE OF OUR EQUATIONS. BUT $V_1 - V_3 = 0$ SO $V_1 = V_3$

∴

$$\begin{array}{c} \\ \text{①} \\ \text{②} \\ \text{③} \end{array}\begin{array}{c} \\ \left[\begin{matrix} i_1 - i_2 \\ i_2 - i_{ARB} \\ i_{ARB} \end{matrix}\right] \end{array} = \left[\begin{array}{ccc} \text{①} & \text{②} & \text{③} \\ (G_1+G_2+sC_1) & -(G_2+sC_1) & 0 \\ -(G_2+sC_1) & (G_2+G_3+G_4+sC_1) & -G_4 \\ 0 & -G_4 & (G_4+sC_2) \end{array}\right]\begin{array}{c} \\ \left[\begin{matrix} v_1 \\ v_2 \\ v_1 \end{matrix}\right] \end{array}$$

- SINCE THE TERMS IN COLUMN 3 OF $[y]$ ARE ALL MULTIPLIED BY V_1 THEN THE THIRD COLUMN CAN BE ELIMINATED BY ADDING THIS TO COLUMN 1.

$$\begin{array}{c} \\ \text{①} \\ \text{②} \\ \text{③} \end{array}\begin{array}{c} \\ \left[\begin{matrix} i_1 - i_2 \\ i_2 - i_{ARB} \\ i_{ARB} \end{matrix}\right] \end{array} = \left[\begin{array}{cc} (1,3) & \text{②} \\ (G_1+G_2+sC_1) & -(G_2+sC_1) \\ -(G_2+sC_1+G_4) & (G_2+G_3+G_4+sC_1) \\ (G_4+sC_2) & -G_4 \end{array}\right]\left[\begin{matrix} v_{1,3} \\ v_2 \end{matrix}\right]$$

- SINCE THERE ARE ONLY TWO UNKNOWNS, WE ONLY NEED TWO EQUATIONS. BY ADDING ROWS 2 AND 3 WE CAN ELIMINATE THE CURRENT i_{ARB}

$$\begin{array}{c} \\ \text{①} \\ (2,3) \end{array}\begin{array}{c} \\ \left[\begin{matrix} i_1 - i_2 \\ i_2 \end{matrix}\right] \end{array} = \left[\begin{array}{cc} (1,3) & \text{②} \\ (G_1+G_2+sC_1) & -(G_2+sC_1) \\ -(G_2+sC_1)+sC_2 & (G_2+G_3+sC_1) \end{array}\right]\left[\begin{matrix} v_{1,3} \\ v_2 \end{matrix}\right]$$

- THUS ADDING A NULLATOR BETWEEN NODES 1 AND 3 RESULTED IN ADDING COLUMNS 1 AND 3 OF THE PASSIVE $[y]$ MATRIX. ADDING A NORATOR BETWEEN NODES 2 AND 3 RESULTED IN ADDING ROWS 2 AND 3 OF THE PASSIVE CIRCUIT $[I]$ VECTOR AND $[y]$ MATRIX.

- THERE ARE TWO OTHER CONDITIONS TO CONSIDER. IF A NULLATOR IS GROUNDED THEN THE UNGROUNDED NODE IS NOW FORCED TO ZERO VOLTS. THIS RESULTS IN A COLUMN OF $[y]$ BEING MULTIPLIED BY ZERO. THIS IS THE SAME AS DELETING THAT COLUMN. IF A NORATOR IS GROUNDED THEN i_{ARB} ONLY APPEARS IN ONE ROW OF $[\phi]$. THUS WE CAN DELETE THAT ROW.

1) NULLATOR - NORATOR NODAL ANALYSIS ALGORITHM

IF AN $(N+1)$ NODE CIRCUIT IS COMPOSED OF RLC ELEMENTS, INDEPENDENT CURRENT SOURCES, M-NULLATORS AND M-NORATORS, THEN THE FOLLOWING STEPS MAY BE USED TO ANALYZE THE CIRCUIT.

a) WITH ALL OF THE NULLATORS AND NORATORS OPEN CIRCUITED, FORM THE NODAL EQUATIONS USING THE ALGORITHM OF CH. 1, P6. WE HAVE

$$[\phi] = [y]_{N\times N}[V]$$

b) FOR A NULLATOR BETWEEN NODES D AND J, ADD COLUMN J TO D OF $[y]_{N\times N}$. DELETE COLUMN J FROM $[y]_{N\times N}$ AND DELETE V_J FROM $[V]$. RELABEL V_D AS $V_{D,J}$ AND RELABEL COLUMN D AS D, J.

c) FOR A NULLATOR BETWEEN NODES E AND GROUND, DELETE COLUMN E FROM $[y]_{N \times N}$ AND DELETE V_E FROM $[V]$.

d) FOR A NORATOR BETWEEN NODES F AND H, ADD ROW H TO F OF $[y]_{N \times (N-1)}$ AND $[\mathcal{I}]$. DELETE ROW H FROM $[y]_{N \times (N-1)}$ AND $[\mathcal{I}]$. RELABEL ROW F AS F,H.

e) FOR A NORATOR BETWEEN NODES G AND GROUND, DELETE ROW G FROM $[y]_{N \times (N-1)}$ AND $[\mathcal{I}]$.

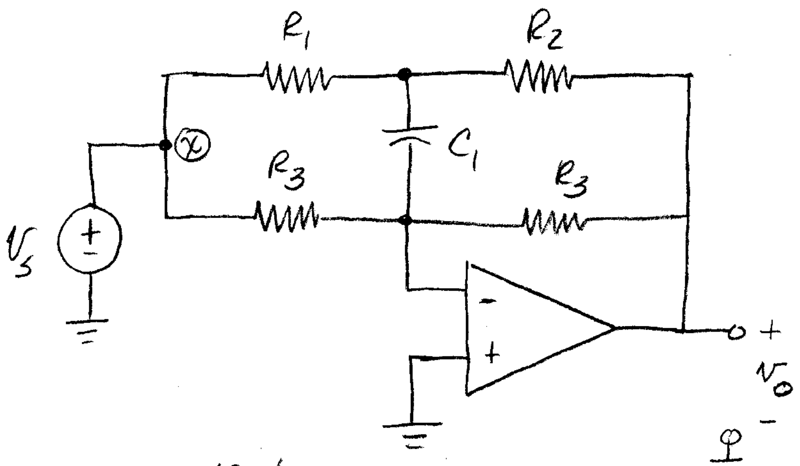
f) REPEATING STEPS b-e m-TIMES RESULTS IN $[y]_{N \times N}$ BEING REDUCED BY m-ROWS AND m-COLUMNS, THAT IS, $y_{(N-m) \times (N-m)}$. THUS

$$[\mathcal{I}] = [y]_{(N-m) \times (N-m)} [V]$$

WHERE
$$[V] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_{N-m} \end{bmatrix}$$

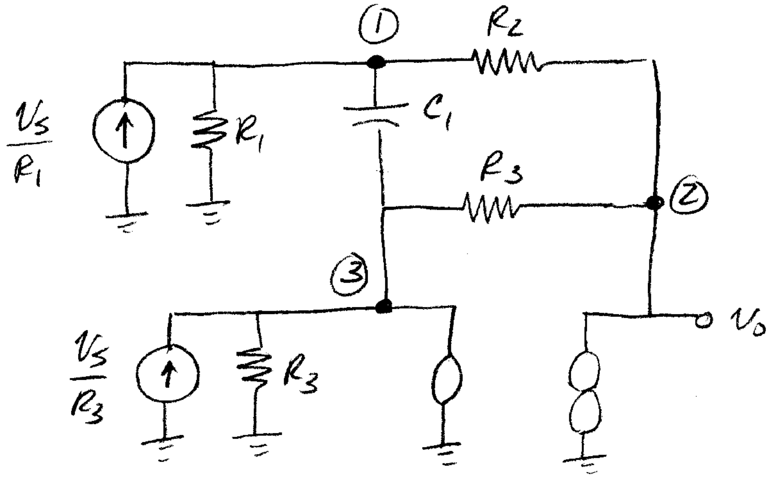
IS THE UNKNOWN COLUMN VECTOR OF NODE VOLTAGES.

2) EXAMPLE : TREBLE TONE CONTROL



SOLVE FOR V_o / V_s

SOLUTION: PUSHING V_s THROUGH THE NODE x AND DOING A SOURCE TRANSFORMATION



a) WITH THE NULLATOR AND NORATOR OPEN CIRCUITED

$$\begin{array}{c} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array}\begin{bmatrix} G_1 V_s \\ 0 \\ G_3 V_s \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \textcircled{1} & \textcircled{2} & \textcircled{3} \end{array} \\ \begin{bmatrix} (G_1 + G_2 + sC_1) & -G_2 & -sC_1 \\ -G_2 & (G_2 + G_3) & -G_3 \\ -sC_1 & -G_3 & 2G_3 + sC_1 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

c) DELETE COLUMN 3 AND REMOVE V_3 FROM $[V]$ SINCE THERE IS A NULLATOR AT NODE 3 TO GROUND.

$$\begin{array}{c} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \begin{bmatrix} G_1 V_S \\ 0 \\ G_3 V_S \end{bmatrix} = \begin{array}{cc} \textcircled{1} & \textcircled{2} \\ \left[\begin{array}{c} (G_1+G_2+sC_1) \\ -G_2 \\ -sC_1 \end{array}\right. & \left.\begin{array}{c} -G_2 \\ (G_2+G_3) \\ -G_3 \end{array}\right] \end{array} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

e) DELETE ROW 2 FROM $[\mathcal{I}]$ AND $[\mathcal{Y}]$ DUE TO THE NORATOR

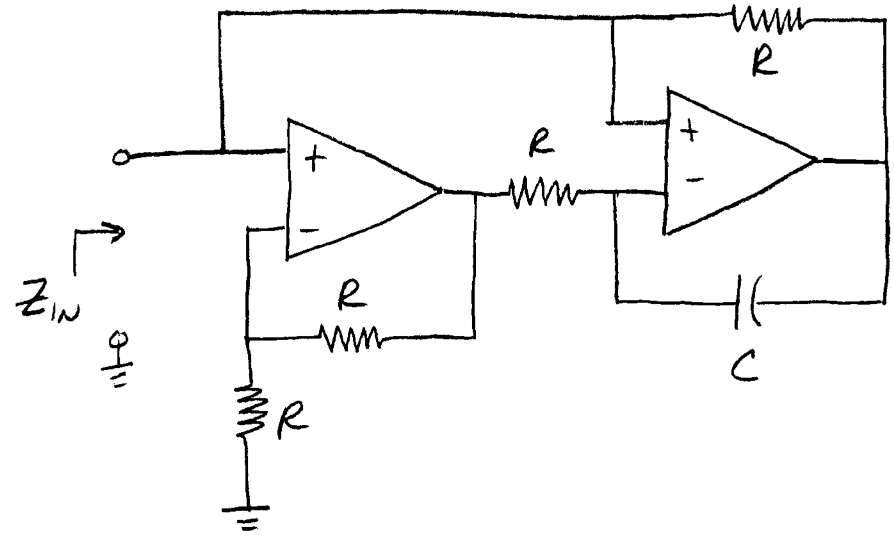
$$\begin{array}{c} \\ \textcircled{1} \\ \textcircled{3} \end{array} \begin{bmatrix} G_1 V_S \\ G_3 V_S \end{bmatrix} = \begin{array}{cc} \textcircled{1} & \textcircled{2} \\ \left[\begin{array}{c} (G_1+G_2+sC_1) \\ -sC_1 \end{array}\right. & \left.\begin{array}{c} -G_2 \\ -G_3 \end{array}\right] \end{array} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$V_2 = \frac{\begin{vmatrix} G_1+G_2+sC_1 & G_1 V_S \\ -sC_1 & G_3 V_S \end{vmatrix}}{\begin{vmatrix} G_1+G_2+sC_1 & -G_2 \\ -sC_1 & -G_3 \end{vmatrix}}$$

$$\frac{V_2}{V_S} = \frac{G_3\left[G_1+G_2+sC_1\right] + sC_1 G_1}{-G_3\left[G_1+G_2+sC_1\right] - sC_1 G_2} \quad \& \quad V_0 = V_2$$

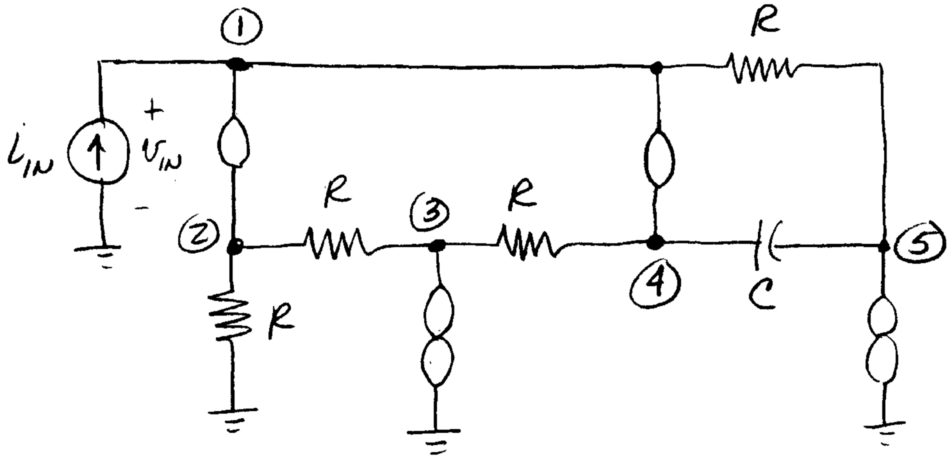
$$\therefore \quad \frac{V_0}{V_S} = -\frac{sC_1\left[G_1+G_3\right] + G_3\left[G_1+G_2\right]}{sC_1\left[G_2+G_3\right] + G_s\left[G_1+G_2\right]}$$

3) SIMULATED IMPEDANCE



FIND Z_{IN}

SOLUTION: WE CAN FIND Z_{IN} BY APPLYING A CURRENT i_{IN} AND SOLVING FOR v_{IN}



a) WITH THE NULLATORS AND NORATORS OPEN CIRCUITED

$$\begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \\ \textcircled{5} \end{array} \begin{bmatrix} i_{IN} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{array}{ccccc} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} \end{array} \\ \begin{bmatrix} G & 0 & 0 & 0 & -G \\ 0 & 2G & -G & 0 & 0 \\ 0 & -G & 2G & -G & 0 \\ 0 & 0 & -G & sC+G & -sC \\ -G & 0 & 0 & -sC & sC+G \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

b) & e) ADD COLUMN 4 TO 1; DELETE ROW 5

$$\begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{array} \begin{bmatrix} i_{IN} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} (1,4) & \textcircled{2} & \textcircled{3} & \textcircled{5} \end{array} \\ \begin{bmatrix} G & 0 & 0 & -G \\ 0 & 2G & -G & 0 \\ -G & -G & 2G & 0 \\ sC+G & 0 & -G & -sC \end{bmatrix} \end{array} \begin{bmatrix} V_{1,4} \\ V_2 \\ V_3 \\ V_5 \end{bmatrix}$$

b) & e) ADD 2 TO COLUMN 1,4; DELETE ROW 3

$$\begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{4} \end{array} \begin{bmatrix} i_{IN} \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} (1,2,4) & \textcircled{3} & \textcircled{5} \end{array} \\ \begin{bmatrix} G & 0 & -G \\ 2G & -G & 0 \\ sC+G & -G & -sC \end{bmatrix} \end{array} \begin{bmatrix} V_{1,2,4} \\ V_3 \\ V_5 \end{bmatrix}$$

- SOLVING FOR $V_{1,2,4} \Rightarrow V_1 = V_2 = V_4 = V_{IN}$

$$V_{IN} = \frac{\begin{vmatrix} i_{IN} & 0 & -G \\ 0 & -G & 0 \\ 0 & -G & -sC \end{vmatrix}}{\begin{vmatrix} G & 0 & -G \\ 2G & -G & 0 \\ sC+G & -G & -sC \end{vmatrix}}$$

$\leftarrow$ EXPAND DOWN COLUMN 1 (numerator)

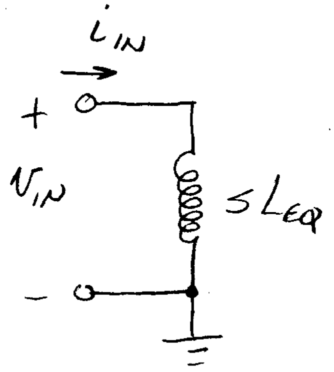
$\leftarrow$ EXPAND DOWN COLUMN 2 (denominator)

$$V_{IN} = \frac{i_{IN}\left[(-G)(-sC)-(0)(-G)\right]}{-G\left[(G)(-sC)-(-G)(sC+G)\right]-(-G)\left[G(0)-(-G)(2G)\right]}$$

$$= \frac{i_{IN}\,GsC}{G^2sC - G^2sC - G^3 + 2G^3} = i_{IN}\frac{sC}{G^2} = i_{IN}\,sCR^2$$

$$\therefore\ Z_{IN} = \frac{V_{IN}}{i_{IN}} = sCR^2$$

- THIS IS THE FORM OF AN INDUCTANCE sL_{EQ}



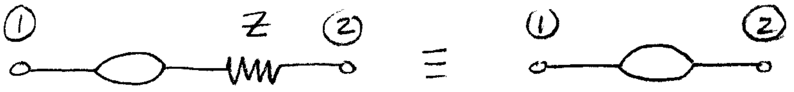
WHERE $L_{EQ} = CR^2$

- IF $C = 0.1\mu F$ AND $R = 10K\Omega$ THEN

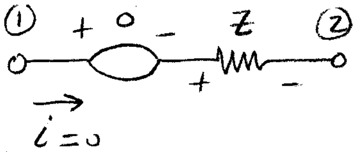
$$L_{EQ} = (0.1\mu)(10K)^2 = 10\,H$$

C) EQUIVALENCE RELATIONSHIPS

1) NULLATOR SERIES EQUIVALENCE

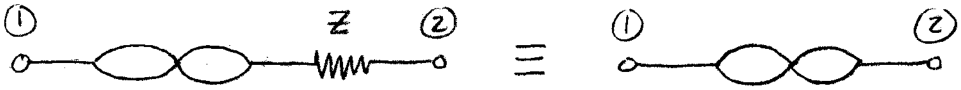


PROOF:

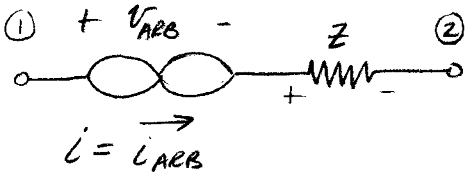


$$\left.\begin{aligned} \therefore\ v_{12} &= 0 + 0 \cdot Z = 0 \\ \&\ i &= 0 \end{aligned}\right\} = \text{NULLATOR}$$

2) NORATOR SERIES EQUIVALENCE

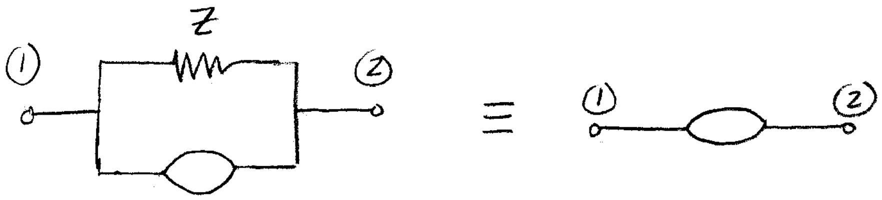


PROOF:

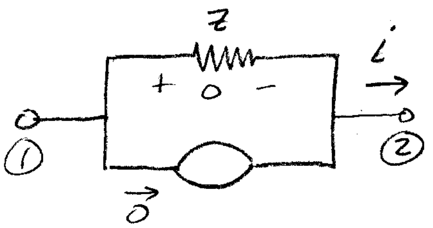


$$\left.\begin{aligned} \therefore\ v_{12} &= v_{ARB} + i_{ARB}\, Z \\ &= \text{NEW } v_{ARB} \\ \&\ i &= i_{ARB} \end{aligned}\right\} \text{NORATOR}$$

3) NULLATOR PARALLEL EQUIVALENCE

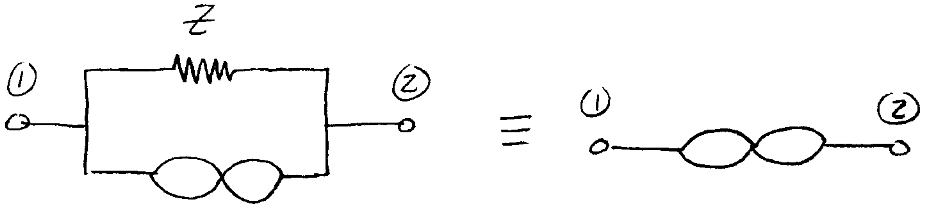


PROOF:

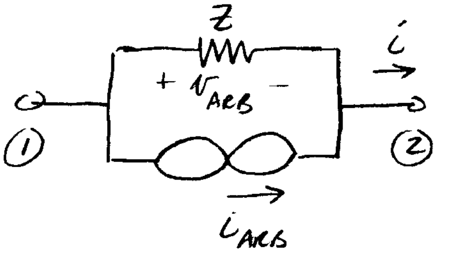


$$\left.\begin{aligned} \therefore\ i &= \frac{0}{Z} + 0 = 0 \\ \&\ v_{12} &= 0 \end{aligned}\right\} \text{NULLATOR}$$

4) NORATOR PARALLEL EQUIVALENCE

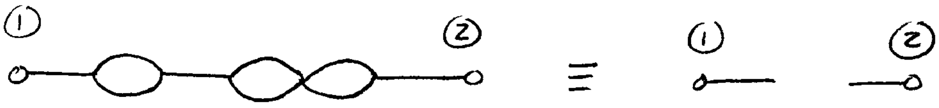


PROOF:

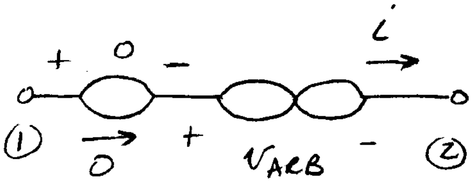


$$\left.\begin{aligned} \therefore\ i &= \frac{v_{ARB}}{Z} + i_{ARB} \\ &= \text{NEW } i_{ARB} \\ \text{\& } v_{12} &= v_{ARB} \end{aligned}\right\} \text{NORATOR}$$

5) OPEN CIRCUIT EQUIVALENCE

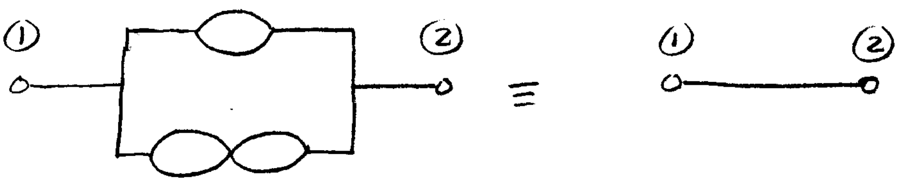


PROOF:



$$\therefore \left.\begin{aligned} i &= 0 \\ \text{\&} \\ v_{12} &= 0 + v_{ARB} \end{aligned}\right\} \text{OPEN CIRCUIT}$$

6) SHORT CIRCUIT EQUIVALENCE

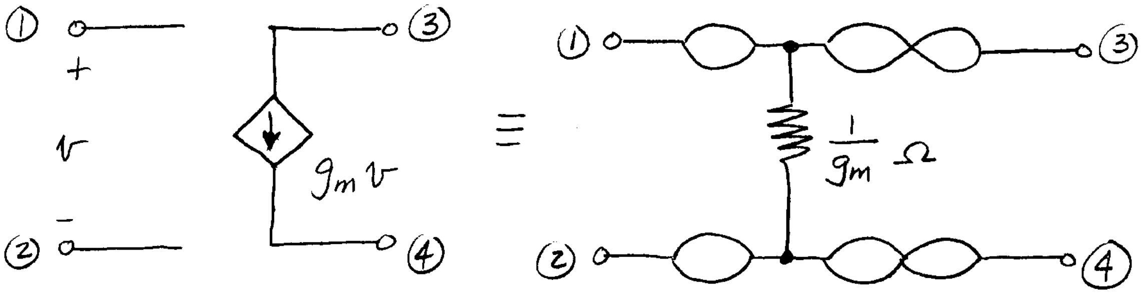


PROOF:

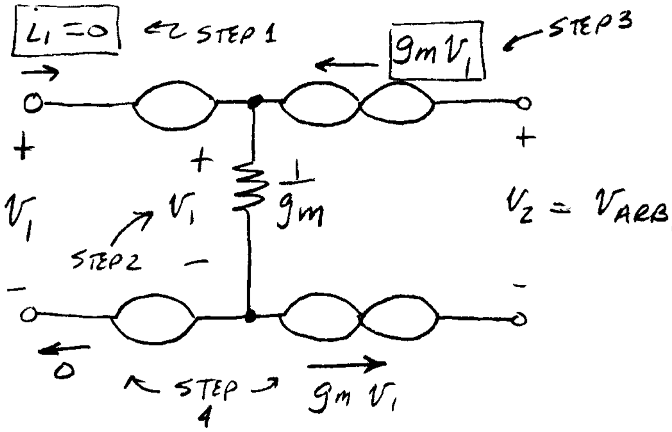
$$v_{12} = 0\ ,\quad i = 0 + i_{ARB} \ \Big\}\ \text{SHORT CIRCUIT}$$

D) CONTROLLED SOURCE MODELS

1) VOLTAGE-CONTROLLED CURRENT SOURCE (VCCS)

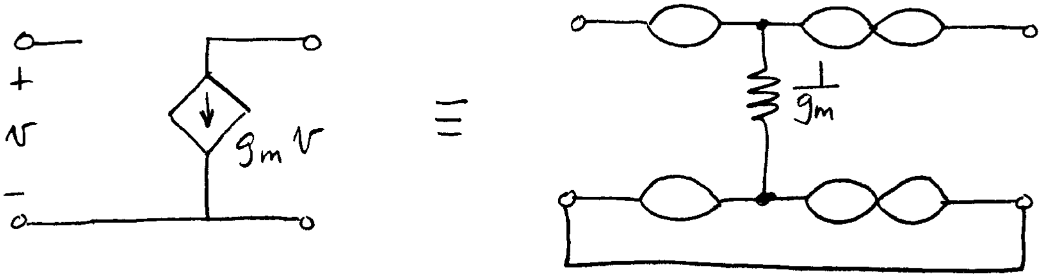


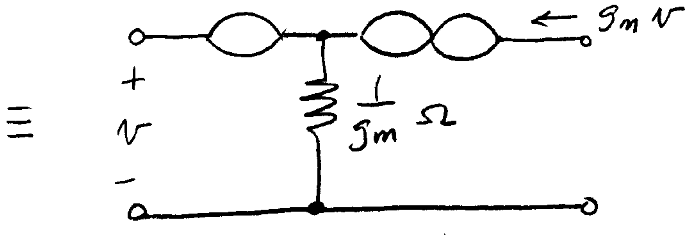
PROOF:



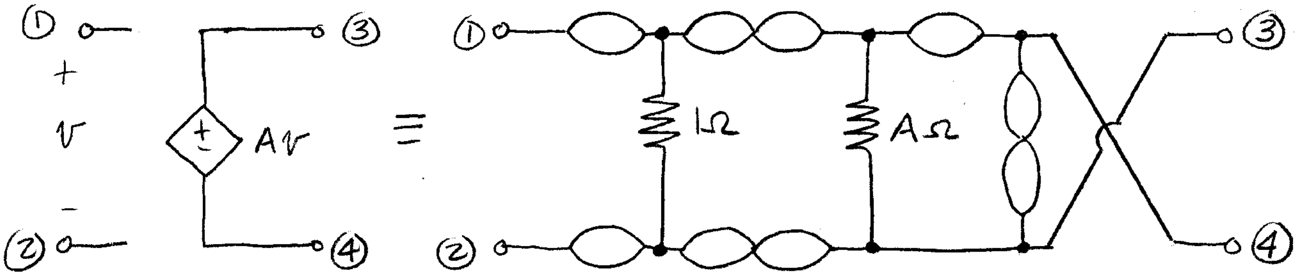
$V_2 = V_{ARB_1} + V_1 + V_{ARB_2} = \boxed{V_{ARB}}$

— NOTE: IF WE HAVE A 3-TERMINAL VCCS





2) VOLTAGE-CONTROLLED VOLTAGE SOURCE (VCVS)



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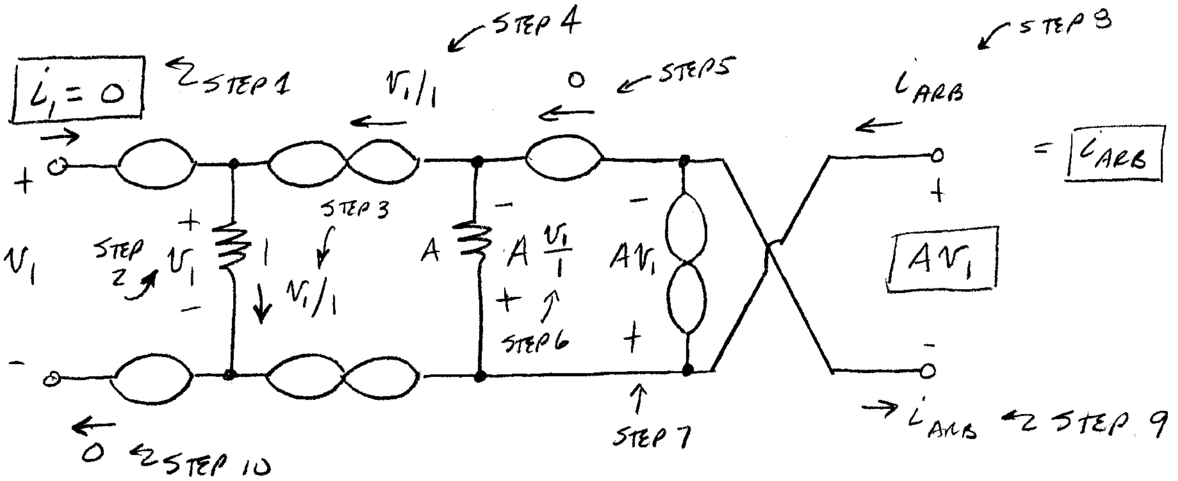
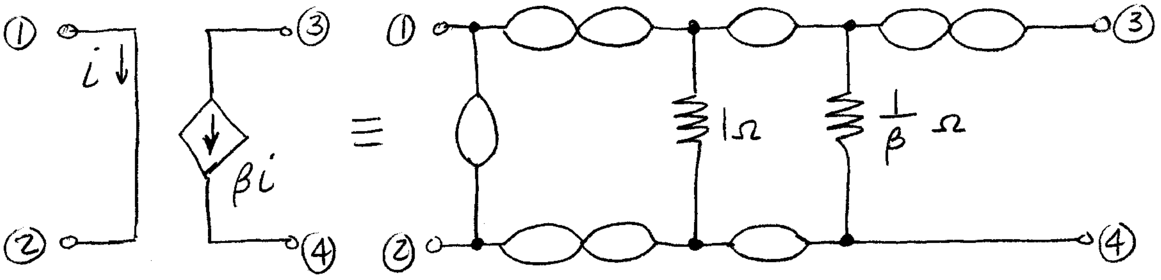


3) CURRENT-CONTROLLED CURRENT SOURCE (CCCS)



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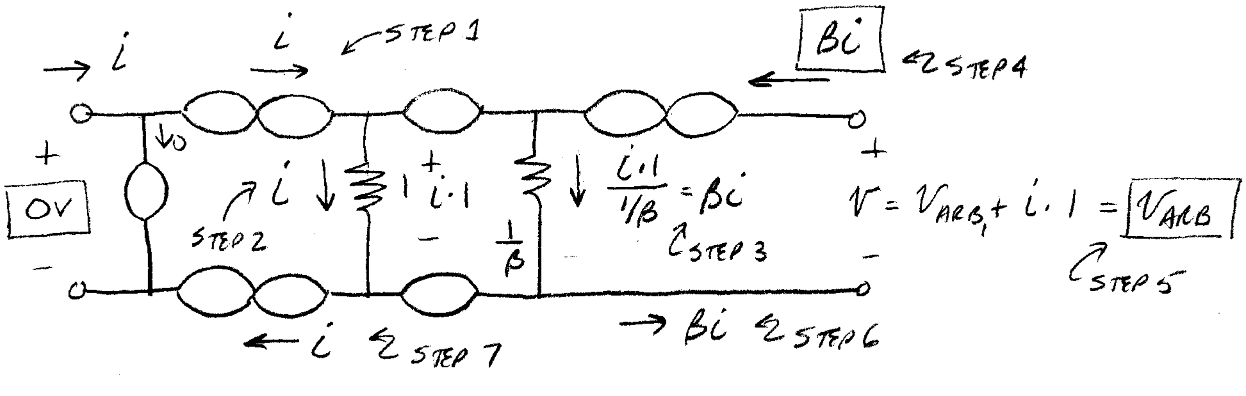
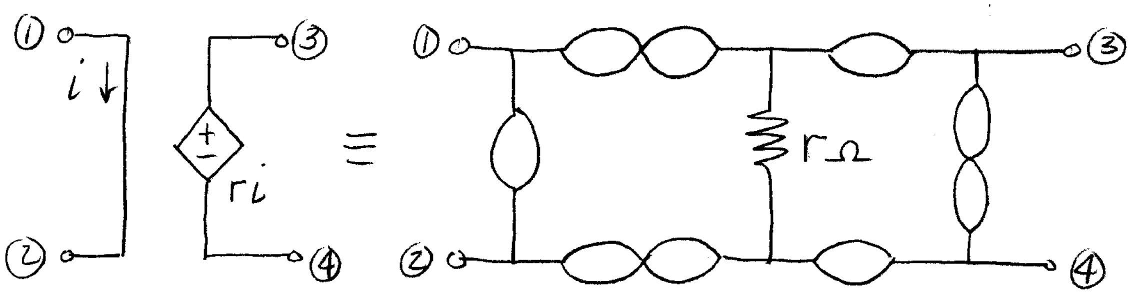


4) CURRENT-CONTROLLED VOLTAGE SOURCE (CCVS)



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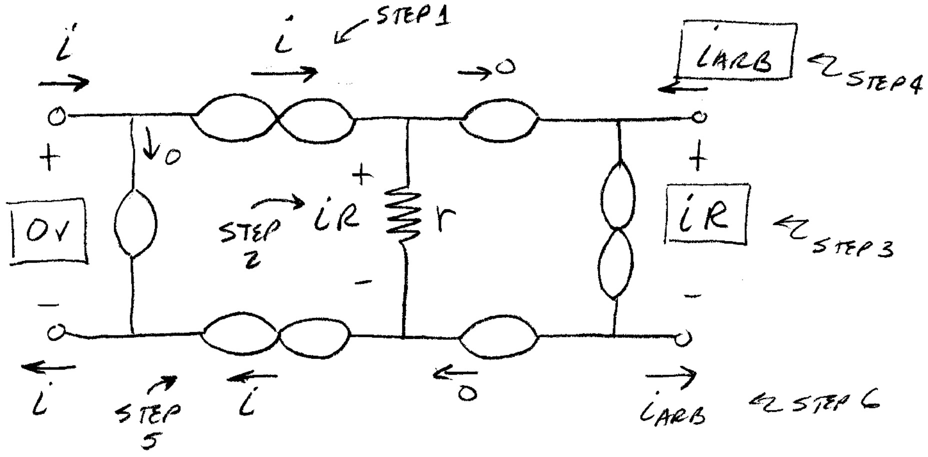
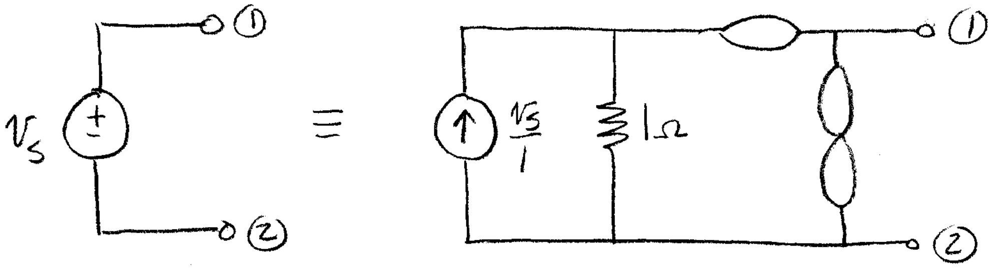


E) SYMBOLIC SPICE

- PERFORMING THE NODAL ANALYSIS ALGORITHM WITH MANY NULLATORS AND NORATORS IS VERY TEDIOUS AND IT IS BEST TO TURN TO SOFTWARE TO KEEP TRACK OF OPERATIONS

- THE NODAL ANALYSIS ALGORITHM DOESN'T INCLUDE VOLTAGE SOURCES. IS IT POSSIBLE TO SIMULATE A VOLTAGE SOURCE WITH A CURRENT SOURCE AND NULLATORS - NORATORS ?

1) SIMULATED VOLTAGE SOURCE



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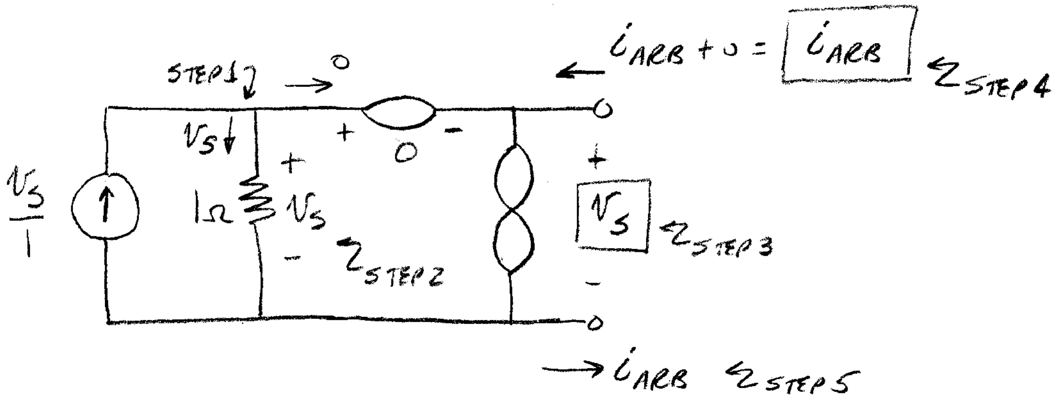
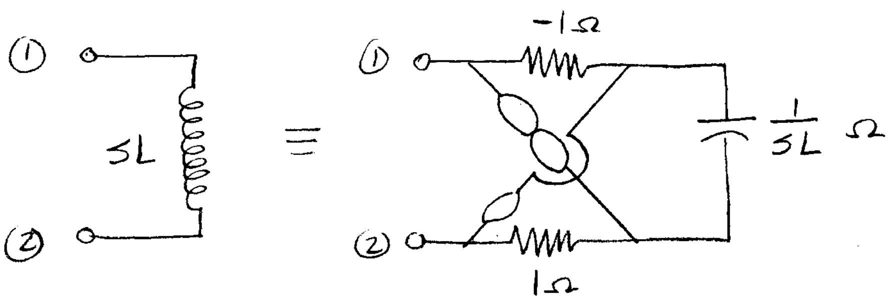


— THE NULLATOR - NORATOR NODAL ANALYSIS ALGORITHM USES ADMITTANCES IN FORMING THE $[y]$ - MATRIX. INDUCTANCES HAVE AN ADMITTANCE WHICH IS $1/sL$. DIVISION AND COMMON FACTORS ARE VERY DIFFICULT TO DEAL WITH AND TEND TO EXPLODE SYMBOLIC RESULTS. IS IT POSSIBLE TO SIMULATE AN INDUCTANCE USING A CAPACITANCE AND NULLATORS - NORATORS?

2) SIMULATED INDUCTANCE



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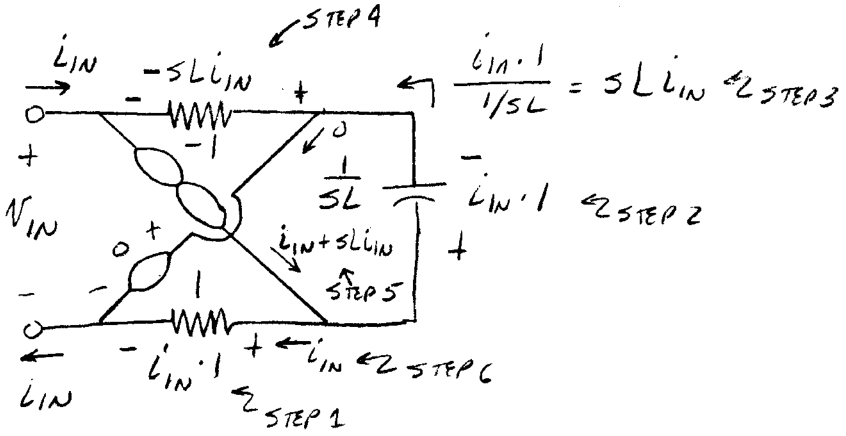

$$v_{IN} + (-sL\,i_{IN}) = 0 \Rightarrow \boxed{\frac{v_{IN}}{i_{IN}} = sL}$$

- THIS CIRCUIT BASICALLY TAKES Z_X AND MAKES THE INPUT IMPEDANCE $1/Z_X$.

- IN ORDER TO REALIZE A VCVS WE NEED A CONDUCTANCE OF $1/A$. THE ABOVE CIRCUIT CAN BE USED TO SIMULATE A RESISTANCE, A, USING A CONDUCTANCE OF A.

- A SIMILAR COMMENT FOR A CCVS.

3) SSPICE PROGRAM

- THE SSPICE PROGRAM USES CURRENT SOURCES, CONDUCTANCES, CAPACITANCES AND NULLATORS-NORATORS TO BUILD A PARTS SET EQUIVALENT TO THE SPICE PROGRAM. THE USER IS UNAWARE OF THIS MODELING TECHNIQUE IN THAT SSPICE HAS THE SAME PARTS SPICE HAS IN THE FREQUENCY DOMAIN (AC) ANALYSIS.

4) THE SOFTWARE CAN BE FOUND AT

www.egr.msu.edu/~wierzba

JUST CLICK ON THE SSPICE BUTTON.

5) SSPICE MANUAL

- THE SSPICE MANUAL CONTAINS DEFINITIONS OF MODELS, ANALYSIS OPTIONS AND THE BASICS OF RUNNING THE PROGRAM.

- SEE THE SSPICE MANUAL VERSION 1.0 FOR MORE DETAILS.