Symbolic SPICE

Symbolic SPICE™

Circuit Analyzer and Approximator

Application Note

AN-003:

Notch Filter Using a Simulated Inductor

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A) Introduction

The schematic shown in Fig. 1 is a notch filter which uses a series resonant circuit [1-2]. At the resonant frequency [3] of $1/(2\pi L_1 C_1)$, L_1 and C_1 form a short circuit. The resulting circuit is a differential amplifier with a difference of inputs equal to zero resulting in an output voltage of zero. Off resonance L_1 and C_1 act like an open circuit which forces $V_1 = V_4 = V_3 = V_2$. This produces a gain with a magnitude of one. Thus there is a notch at the frequency of $f_o = 1/(2\pi L_1 C_1)$.

A Symbolic SPICE™ input file, **`notch.cir`**, is given in Table 1. This file uses Symbolic SPICE's definition [4] of an ideal op-amp which begins with the letters `XOA`.

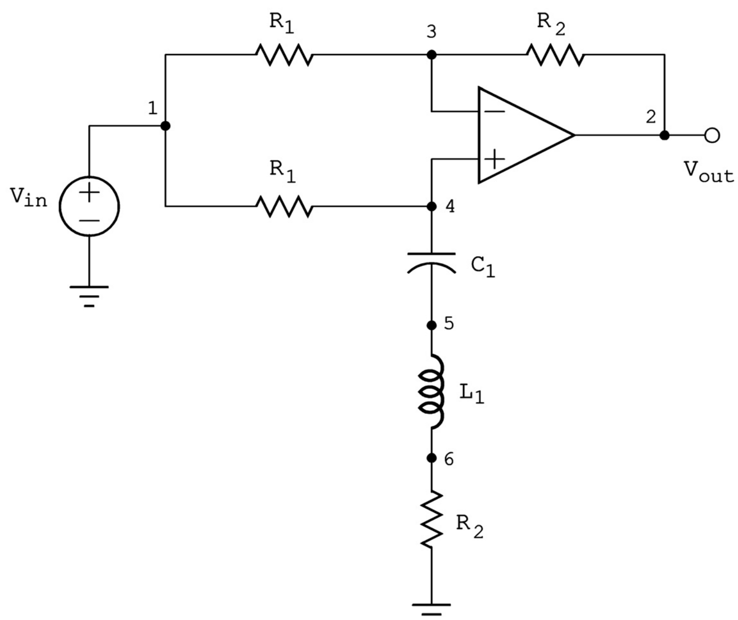

Figure 1. Notch filter

Table 1. Symbolic SPICE input file

```
Notch Filter
VS 1 0 AC 1
R1 1 3
R2 3 2
R1 1 4
R2 6 0
C1 4 5
L1 5 6
XOA 4 3 2
.END
```


B) Running Symbolic SPICE

Running [4] the input file shown in Table 1, the following are the prompts. The user responses are shown in bold:

```
Symbolic SPICE - Circuit Analyzer and Approximator
Demo Version 3.1
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```

```
INPUT FILE NAME [.cir] : notch

OUTPUT FILENAME [notch.det] : (hit enter)

Determinant string sorted according to orders of some variable? (y/n) : n

Numerical evaluation of the results? (y/n) : n

Discard terms if their magnitude falls below a threshold? (y/n) : n

Check and solve for second order filter functions? (y/n) : y

FILTER FUNCTION FILE NAME [notch.fun] : (hit enter)

Solve for a variable or expression? (y/n) : y

Available Unknowns:
V1 V2 V3 V5 V6 V8

*Ignore nodes 7 and higher if present. They are used for internal numbering.

Valid Operators: +, -, *, /, ( ), { }, [ ]

Equation: v2

Solve for another variable or expression? (y/n) : n
```

C) Symbolic SPICE Determinant Output

The output file **notch.det** listed in Table 2 is the matrix written by Symbolic SPICE and the transfer function requested.

Table 2. Symbolic SPICE Output File **notch.det**

```
Notch Filter

[0  ] [-G1       -G2       G2+G1     0         0         0           ][V1 ]
[0  ] [-G1       0         sC1+G1    -sC1      0         0           ][V2 ]
[0  ]=[0         0         -sC1      sC1-1     0         1           ][V3 ]
[0  ] [0         0         0         -sL1      G2+1      sL1-1       ][V5 ]
[1  ] [1         0         0         0         0         0           ][V6 ]
[0  ] [0         0         0         sL1+1     -1        -sL1        ][V8 ]

*Ignore nodes 7 and higher if present. They are used for internal numbering.

Numerator of: v2

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:
```

```
 + sL1*sC1*G2*G1

s**0 terms:

 + G2*G1

************************************************

Denominator of: v2

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sL1*sC1*G2*G1

s**1 terms:

 + sC1*G2 + sC1*G1

s**0 terms:

 + G2*G1

*************************************************
```

D) Symbolic SPICE Filter Function Output

Symbolic SPICE will examine any transfer function to determine if its denominator is second order in the Laplace operator *s*. If this is the case then it finds the parameters associated with the different filter functions. Symbolic SPICE finds formulas for Q_o, ω_o and H_i [4].

The output file **notch.fun** listed in Table 3 contains the filter parameters solved for by Symbolic SPICE.

Table 3. Symbolic SPICE Output File **notch.fun**

```
Notch Filter

SECOND ORDER FILTER PARAMETERS:

Qo is:

( + G1*G2)*SQRT{ + L1}
-------------------
( + G2 + G1)*SQRT{ + C1}

Wo**2 is:

( + 1)
--------
( + L1*C1)

****************************************
```

```
There exists a BAND STOP filter for : v2

BAND STOP GAIN (Hbs) is:

( + 1)
--------
 + 1

BAND STOP FREQUENCY (Wz**2) is:

( + 1)
--------
( + L1*C1)

****************************************
```

Symbolic SPICE's format is a collection of strings of symbols. This is usually not how most people view equations, so you may need to do some minor editing. Thus we have:

$$Q_o = \frac{G_1 G_2}{G_2 + G_1}\sqrt{\frac{L_1}{C_1}} = \frac{1}{R_2 + R_1}\sqrt{\frac{L_1}{C_1}}$$

$$\omega_o = \omega_z = \frac{1}{\sqrt{L_1 C_1}}$$

The results show that the band-stop' s frequency ω_z is the same as the center frequency ω_o. This implies that our band-stop filter is a notch filter. Also the gain off the notch frequency, H_{bs}, is one.

Another useful formula is that of the -3 dB bandwidth (radians/sec)

$$\text{Bandwidth} = \frac{\omega_o}{Q_o} = (R_2 + R_1)\sqrt{\frac{C_1}{L_1}}\frac{1}{\sqrt{L_1 C_1}} = \frac{(R_2 + R_1)}{L_1}$$

E) Simulated Inductor

Most active filters avoid the use of inductors because of mutual inductance effects with the power supply transformer or other inductive circuit elements. Since there is a series resistor with the inductor of Fig. 1, we can use a lossy simulated inductor [1]. This is shown in Fig. 2. The Symbolic SPICE input file, **`lossy_L.cir,`** is given in Table 4. *Note that when a current source of 1 A is used as an input then each node voltage is a transfer impedance and more specifically the input node voltage is the input impedance.*

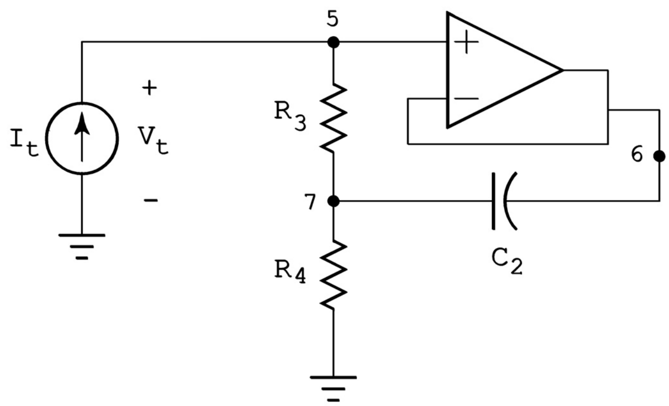

Figure 2. Lossy simulated inductor

Table 4. Symbolic SPICE input file **lossy_L.cir**

```
SIMULATED INDUCTOR
It 0 5 AC 1
R3 5 7
R4 7 0
C2 6 7
XOA 5 6 6
.END
```

Running the input file shown in Table 4, the following are the prompts. The user responses are shown in bold:

```
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```

INPUT FILE NAME [.cir] : **lossy_L.cir**

OUTPUT FILENAME [lossy_L.det] : **(hit enter)**

Determinant string sorted according to orders of some variable? (y/n) : **n**

Numerical evaluation of the results? (y/n) : **n**

Check and solve for second order filter functions? (y/n) : **n**

Solve for a variable or expression? (y/n) : **y**

Available Unknowns:
V5 V7

*Ignore nodes 8 and higher if present. They are used for internal numbering.

Valid Operators: +, -, *, /, ( ), { }, [ ]

Equation: **v5**

Solve for another variable or expression? (y/n) : **n**

F) Symbolic SPICE Determinant Output

The output file **`lossy_L.det`** listed in Table 5 is the matrix written by Symbolic SPICE and the transfer function requested.

Table 5. Symbolic SPICE Output File **`lossy_L.det`**

```
SIMULATED INDUCTOR

[1  ]=[G3                              -G3                              ][V5 ]
[0  ] [-sC2-G3                         sC2+G4+G3                        ][V7 ]

*Ignore nodes 8 and higher if present. They are used for internal numbering.

Numerator of: v5

TERMS SORTED ACCORDING TO POWERS OF s

s**1 terms:

 + sC2

s**0 terms:

 + G4 + G3

************************************************

Denominator of: v5

TERMS SORTED ACCORDING TO POWERS OF s

s**0 terms:

 + G4*G3

************************************************
```

After some minor editing, we have

$$Z_{in} = \frac{V_t}{I_t} = \frac{V_5}{1} = \frac{sC_2 + G_4 + G_3}{G_4 G_3} = s\frac{C_2}{G_4 G_3} + \frac{G_4 + G_3}{G_4 G_3} = sC_2 R_4 R_3 + (R_4 + R_3)$$

For our simulated inductor to work in Fig. 1, we want

$$Z_{in} = sL_1 + R_2$$

Thus we have

$$L_1 = C_2 R_4 R_3 \quad \text{and} \quad R_2 = (R_4 + R_3)$$

G) Notch Filter Design

Putting together Figs. 1 and 2, we have the following filter:

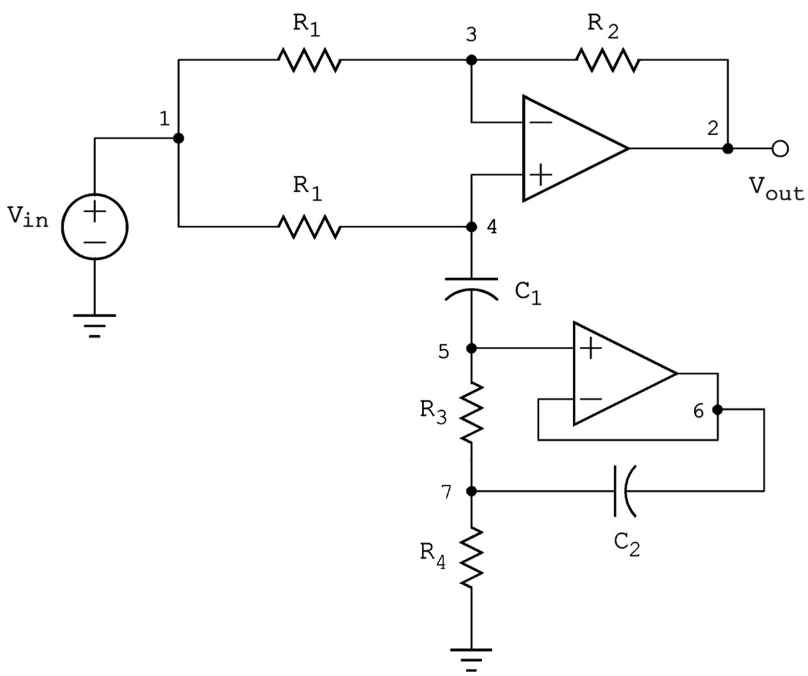

Figure 3. Notch filter using a simulated inductor

We can develop a design procedure using our formulas[1]:

1) Pick $R_3 = R_4 = R$ to be a standard value

2) Let $R_1 = R_2 = 2\ R$

3) $L_1 = (R_2 + R_1)Q_o\ /\omega_o = C_2 R_4 R_3$ then

$C_2 = (R_2 + R_1)Q_o\ /(R_4 R_3\ \omega_o) = 4\ Q_o\ /\ (R\ \omega_o)$

4) $C_1 = 1/(L_1\ \omega_o^2) = 1\ /(C_2\ R^2\ \omega_o^2)$

Suppose we design a notch for $f_o = 120$ Hz with a Q_o of 5 then

1) Pick $R_3 = R_4 = R = 10\text{k}\ \Omega$

2) Let $R_1 = R_2 = 2\ R = 20\text{k}\ \Omega$

3) $C_2 = 4\ Q_o\ /\ (R\ \omega_o) = 4 \bullet 5\ /\ (10\text{k} \bullet 2\pi \bullet 120) = 2.6526\mu\ \text{F}$

4) $C_1 = 1\ /(C_2\ R^2\ \omega_o^2) = 1/\ [(2.6526\mu\ (10\text{k} \bullet 2\pi \bullet 120)^2\ ] = 6631\text{p F}$

Let's verify the design with PSpice.

Table 6. PSpice input file for notch filter

```
Notch Filter
VIN 1 0 AC 1
R1 1 3 20K
R2 3 2 20K
R5 1 4 20K
R3 5 7 10K
R4 7 0 10K
C1 4 5 6631P
C2 6 7 2.6526U
X1 4 3 2 OPAMP
X2 5 6 6 OPAMP
.SUBCKT OPAMP 1 2 3
RI 1 2 100MEG
EA 3 0 1 2 100MEG
.ENDS OPAMP
.AC OCT 1000 60 240
.PROBE
.END
```

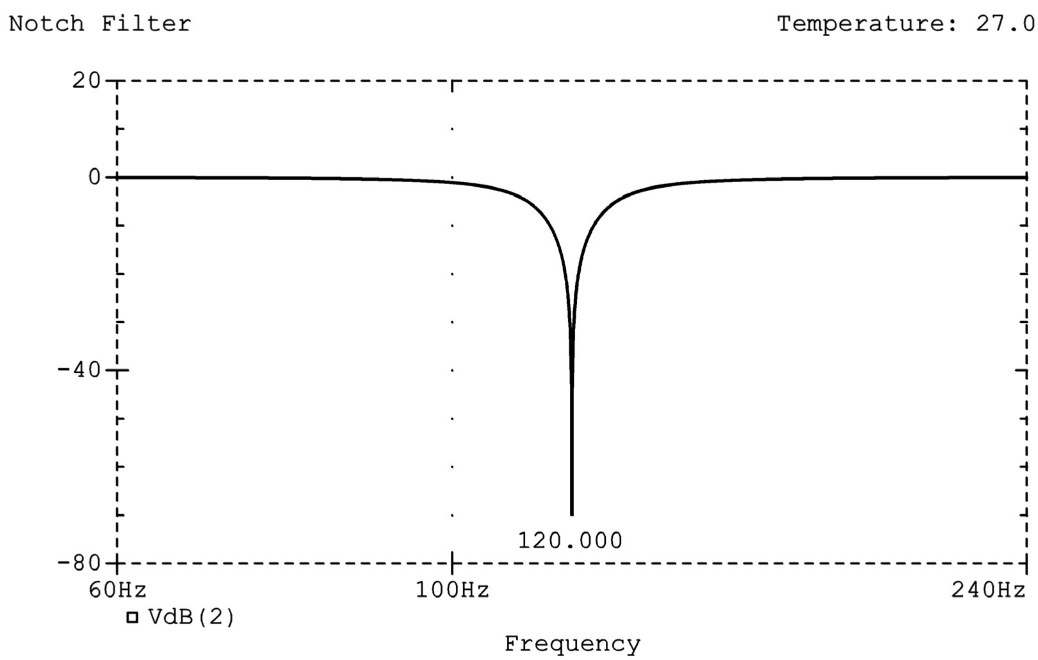

Figure 4. Magnitude of the voltage gain versus frequency

H) Conclusion

Symbolic SPICE can be used to analyze ideal op-amp circuits to find transfer functions and input impedance. Symbolic SPICE can recognize second order transfer functions in the Laplace operator *s*. It can also determine the filter type such as band-stop and then solve for the filter parameters Q_o, ω_o and H_{bs}.

I) References

[1] D. Stout and M. Kaufman, "*Handbook of Operational Amplifier Circuit Design*," McGraw-Hill, pp. 13.1-13.4, 1976.

[2] G. M. Wierzba, *ECE 402: Application of Analog ICs Class Notes,* Ch.2 , pp. 54-57. This e-book is available at http://stores.lulu.com/willowepublishing

[3] G. M. Wierzba, *AN-001: Series Resonant Circuit*. This application note is available at www.willowelectronics.com

[4] G. M. Wierzba, *Symbolic SPICE User Manual*. This e-book is available at http://stores.lulu.com/willowepublishing

